NUMERICAL EVALUATION OF NORMAL STRESS DISTRIBUTION ON SHIP CROSS SECTION – Part 1

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SUMMARY

Design global stresses in ship structures are important aspects to determined dimensioning of plate thicknesses, longitudinals and grillage systems. According to the BKI Rules for Hull the calculation design global stresses can be carried out either using modelling a complete hull in finite element method (FEM) or using prismatic beam analysis on ship cross section. This study presented for generation normal stress distribution (σ_L) on ship cross section using prismatic beam analysis with various loading condition of Vertical Still Water Bending Moment (M_{SW}), Vertical Wave Bending Moment (M_{WV}), Horizontal Wave Bending Moment (M_{WH}), and Wave Torsional Moment (M_{SW}). Calculation of normal stress distribution presented by a detailed step-by-step numerical procedure for evaluating the normal stress of ship cross section. The comparison result shows virtually no difference between the calculated values and commercial software.

1. INTRODUCTION

Ship hull illustrated as structural box girder arranged of stiffened plating. It is subjected to longitudinal bending load produced by distributed hull weight, cargo weight, buoyancy force and wave force. The hull strength against longitudinal bending, shearing and torsional loads is called hull girder strength, which may be the most fundamental strength of a ship structure. This is because maximum stress response takes place on the deck and/or bottom structure. The ship hull may break if the working longitudinal bending moment exceeds the capacity of the cross section.

BKI developed a rules to determined minimum of safety standard for hull girder strength. Which is mentioned in the BKI Rule Vol II, Section 5 [1] about the minimum requirement for global stress component for hull girder assessment in ship structures. In other hand many studies have been done for calculation of hull girder stress component as well as its formulation derivation. These researches has been conducted such as: (a) Vernon and Nadesau [5] that described unified developments of the St-Venant and warping-based on thin-walled beam theories and their application in the torsional analysis of ship structures, (b) Hu [2] described the mathematical derivation of the equation used for the calculation of cross sectional constants and stress distributions of thin-walled sections, and (c) Lue, et al [4] have been proposed a numerical procedure for computing the warping moment of inertia for an arbitrary cold-formed steel open section, which does not need a sophisticated computer software.

Numerical Procedure of normal stress distribution on the arbitrary of ship cross section is very important to be done at the design stage of ship design, especially if the commercial software is not available and expensive. Furthermore, ships are very complex structures with various type of cross section which is very likely composed of a set of single open or closed section even of multi cell or combination. Therefore, the empirical formula to obtain the distribution of normal stress or shear stress on each cross section of the ship due to the vertical bending moment, horizontal bending moment, or torsional moment is rarely found. In this study, a practically step by step numerical procedure to obtain of normal stress distribution is developed for any arbitrary ship cross section. This developed procedure is examined with commercial software.

2. HULL GIRDER STRESS RESPONSE

The analysis of ship hull girder is carrying out by considered ship as a beam composed of plates, stiffener and girders. Because of ratio between the length of ship and it thickness is so large, thin wall theory was developed in order to analyse the stress responses that occurred. The classical theory of thin-walled beams with arbitrary open cross section is based on following assumptions [3]:

1. Equilibrium of wall element:
$$\frac{d}{ds}q + \left(\frac{d}{dz}\sigma\right) \cdot t = 0 \tag{1}$$
2. Compatibility (shear strain):
$$\frac{d}{ds}w + \frac{d}{dz}v = \gamma = 0 \tag{2}$$
3. Tangential displacement (δv) in terms of ξ, η and φ:
$$\frac{\delta v}{\delta z} = \frac{\delta \xi}{\delta z} \cdot \cos(\alpha) + \frac{\delta \eta}{\delta z} \cdot \sin(\alpha) + h_p \cdot \frac{\delta \varphi}{\delta z} \tag{3}$$

Those assumptions would be coupled with resultants between internal and external forces of a cross section. Hence, the equilibrium equation could be described details as follows:

$$\int \sigma \cdot dA = N_z \tag{4}$$
$$\int \sigma \cdot y \cdot dA = M_x \tag{5}$$
$$\int \sigma \cdot x \cdot dA = M_y \tag{6}$$
$$\int \tau \cdot h_p \cdot dA = \int q \cdot h_p \cdot ds = T_p \tag{7}$$
$$\int \tau \cdot \cos(\alpha) \cdot dA = \int q \cdot \cos(\alpha) \cdot ds = V_x \tag{8}$$

$$\int\tau \cdot \sin(\alpha) \cdot dA = \int q \cdot \sin(\alpha) \cdot ds = V_y \qquad (9)$$

2.1. Bending without Twist

The calculation methods of bending without twist on thin wall structures are identified as [3]:

Step 1 - Look for the longitudinal displacement (w)

Using equations (2), $\frac{d}{ds} w = -\left(\frac{d}{dz} v\right)$ and equation (3) with no twist, $\frac{\delta\varphi}{\delta z} = 0$ followed by integration along the s axis, the longitudinal displacement w can be expressed as:

$w = -\xi' \cdot X - \eta' \cdot Y + u_o(z)$

$w' = -\xi'' \cdot X - \eta'' \cdot Y + u_o'(z)$

$\sigma = E \cdot u' = -E \cdot \xi'' \cdot X - E \cdot \eta'' \cdot Y + E \cdot u_o(z)$ (Hooke's Law), u_0 is the constant of integration

Step 2 - Including internal resultans and external force

Using Equation (4), $\int \sigma \cdot dA = 0$

$$E \cdot u_0'(z) = E \cdot \xi'' \cdot \frac{\int X \cdot dA}{A} + E \cdot \eta'' \cdot \frac{\int Y \cdot dA}{A}$$

$$\sigma = -E \cdot \xi'' \cdot X - E \cdot \eta'' \cdot Y + E \cdot \xi'' \cdot \frac{\int X \cdot dA}{A} + E \cdot \eta'' \cdot \frac{\int Y \cdot dA}{A}$$

$$\sigma = -E \cdot \xi'' \cdot \left(X - \frac{\int X \cdot dA}{A}\right) - E \cdot \eta'' \cdot \left(Y - \frac{\int Y \cdot dA}{A}\right) \qquad (10)$$

centre of gravity of section was given by:

$$X_c = \frac{\int X \cdot dA}{A} \qquad (11)$$

$$Y_c = \frac{\int Y \cdot dA}{A} \qquad (12)$$

and $x = X - X_c$ and $y = Y - Yc$ are normalized distance of X and Y to centre of gravity. Then axial stress is expressed as:

$$\sigma = -E \cdot \xi'' \cdot x - E \cdot \eta'' \cdot y \qquad (13)$$

Then equation (13) substitute to equation (5) and (6) can be expressed as:

$$\int \sigma \cdot y \cdot \delta A = -E \cdot \xi'' \cdot \left(\int x \cdot y \cdot \delta A\right) - E \cdot \eta'' \cdot \left(\int y \cdot y \cdot \delta A\right) = M_x$$

$$\int \sigma \cdot x \cdot \delta A = -E \cdot \xi'' \cdot \left(\int x \cdot x \cdot \delta A\right) - E \cdot \eta'' \cdot \left(\int y \cdot x \cdot \delta A\right) = M_y$$

From these equation can be defined moment of inertia as:

$$I_{xx} = \int y \cdot y \cdot dA, \qquad I_{yy} = \int x \cdot x \cdot dA,$$

$$I_{xy} = I_{yx} = \int x \cdot y \cdot dA \qquad (14)$$

By defined a statical moment, $S_y = \int x \cdot t \cdot ds$,.the moments of inertia can be simplified by using integration by parts. Therefore, Equation (14) became as follows [2]:

$$I_{yy} = -\int_0^b S_y \cdot dx, \qquad I_{xx} = -\int_0^b S_x \cdot dy \qquad I_{xy} = -\int_0^b S_x \cdot dy$$

$$I_{yx} = -\int_0^b S_y \cdot dx \qquad (15)$$

Step 3 - Determination of axial stress

The moment curvature relationships of a prismatic beam are expressed as [2]:

$$-E \cdot \xi'' \cdot I_{yx} - E \cdot \eta'' \cdot I_{yy} = M_x$$

$$-E \cdot \xi'' \cdot I_{xx} - E \cdot \eta'' \cdot I_{xy} = M_y$$

By substituting those quantities into Equation (13), the axial stress can be rewritten as:

$$\sigma = \frac{\left(I_{xx} \cdot M_y - M_x \cdot I_{yx}\right)}{\left(-I_{yx}^2 + I_{yy} \cdot I_{xx}\right)} \cdot x + \frac{\left(-I_{yx} \cdot M_y + M_x \cdot I_{yy}\right)}{\left(-I_{yz}^2 + I_{yy} \cdot I_{xx}\right)} \cdot y$$

with $I_{yx} = 0$, $\sigma = \frac{M_y}{I_{yy}} \cdot x$ or $\sigma = \frac{M_x}{I_{xx}} \cdot y$ (16)

2.2. Warping Stress

If a beam only subjected to pure twisting around its Centre of rotation. There is no lateral displacement ξ and η [2]. Therefore, Equation (3) became as follows:

$$\frac{\delta v}{\delta z} = h_p \cdot \frac{\delta\varphi}{\delta z}$$

According to the basic assumption that the shear deformation of the cross section is only caused by the St-Venant shear stresses, with Equation (2) yields:

$$\frac{\delta w}{\delta s} = -h_p \cdot \frac{\delta\varphi}{\delta z} + \frac{q_s}{G \cdot t} = -h_p \cdot \frac{\delta\varphi}{\delta z} + \frac{J}{2 \cdot A \cdot t} \frac{\delta\varphi}{\delta z}$$

With integration along s can be obtained w, σ, and ω as follows:

$$w = -\varphi' \cdot \omega + \varphi' \cdot \omega_2 + w_0(z)$$

$$\sigma = -E \cdot \varphi'' \cdot \omega + E \cdot \varphi'' \cdot \omega_2 + E \cdot w_0(z)$$

$$\omega = \omega_o - \omega_2 - \frac{\int\left(\omega_o - \omega_2\right) dA}{\int dA} \qquad (17)$$

$$\sigma = -E \cdot \varphi'' \cdot \omega$$

ω_o, ω_2, and ω_n are double sectorial area (warping) coordinate of open section, closed section, and normalized warping respectively, with formulation as follows:

$$\omega_o = \int hp \cdot ds \qquad (18)$$

$$\omega_2 = \frac{\oint hp \cdot ds}{\oint (1/t) ds} \cdot \int \frac{1}{t} \cdot ds \qquad (19)$$

$$\omega_n = \frac{\int\left(\omega_o - \omega_2\right) \cdot dA}{\int dA} \qquad (20)$$

The double sectorial area coordinate of each segment were calculated with respect to centre of twist of section. According to Lue et al [4] centre of twist was given by this formulation.

$$x_0 = \frac{\left(I_{\omega x} \cdot I_{yy} - I_{\omega y} \cdot I_{xy}\right)}{I_{xx} \cdot I_{yy} - I_{xy} \cdot I_{yx}} \quad y_0 = \frac{\left(I_{\omega y} \cdot I_{xx} - I_{\omega x} \cdot I_{xy}\right)}{I_{xx} \cdot I_{yy} - I_{xy} \cdot I_{yx}} \quad (21)$$

$$I\dot{\omega}x = I\dot{\omega}xo - I\dot{\omega}xc \qquad I\dot{\omega}y = I\dot{\omega}yo - I\dot{\omega}yc \qquad (22)$$

Where,

Open section → $I\dot{\omega}x_o = \int Sx.d\dot{\omega}_o$ $I\dot{\omega}y_o = \int Sy.d\dot{\omega}_o$

Closed section → $I\dot{\omega}x_c = \int Sx.d\dot{\omega}_2$ $I\dot{\omega}y_c = \int Sy.d\dot{\omega}_2$

Using Equation (7) and integration along s can be calculated of warping moment of inertia (I_ω):

$$T_\omega = \int q \cdot hp \cdot ds = \int q \cdot \frac{\partial \omega}{\partial s} \cdot \partial s = \int \frac{\partial \sigma}{\partial z} \cdot \omega \cdot t \cdot ds$$

$$T_\omega = -E \cdot \varphi''' \int \omega^2 \cdot t \cdot ds \quad \text{, defined } I_{\omega\omega} = \int \omega^2 \cdot t \cdot ds$$

$$T_\omega = -E \cdot \varphi''' \cdot I_{\omega\omega} \qquad (23)$$

By defining $M'_\omega = T_\omega$ can be derived the σ_ω:

$$\varphi'' = \frac{-M_\omega}{E \cdot I_{\omega\omega}}$$

$$\sigma_\omega = -E \cdot \varphi'' \cdot \omega = \frac{M_\omega}{I_{\omega\omega}} \cdot \omega \qquad (24)$$

3. NUMERICAL FORM FOR NORMAL STRESS DISTRIBUTION

The numerical procedures of normal stresses distribution on cross section have been studied by Hu [2] and Lue, et al [4]. In this study a numerical procedure based on previously researches and equations on sub section 2.1 will be summarized. Those procedure globally divided into 3 steps which each procedure will be applied to a segment with nodal coordinates (X_i,Y_i) and (X_j,Y_j) including coordinate of midpoint length of each segment (X_k,Y_k):

Step 1 - Define centre of gravity (X_c, Y_c)

The center of gravity could be calculated with Equation (11) and (12) as follows:

$$Y_c = \frac{\sum_{n=1}^{k} Yk \cdot Ak}{\sum_{n=1}^{k} Ak} \qquad X_c = \frac{\sum_{n=1}^{k} Xk \cdot Ak}{\sum_{n=1}^{k} Ak} \qquad (25)$$

Step 2 - Define the moment of inertia (I_{xx} or I_{xy})

Calculation static moments Sx and Sy each segment of the cross section are given by:

$$\Delta S_x = \frac{A_k\left(Y_i + Y_k\right)}{4} \qquad ; S_x = S_x + \Delta S_x$$

$$\Delta S_y = \frac{A_k\left(X_i + X_k\right)}{4} \qquad ; S_y = S_y + \Delta S_y$$

The moments of inertia of the cross section can be found as:

$$I_{xx} = \sum_{n=1}^{Seg} -\frac{\Delta X_k}{6}\left[S_x(i) + 4 \cdot S_x(k) + S_x(j)\right]$$

$$I_{yy} = \sum_{n=1}^{Seg} -\frac{\Delta Y_k}{6}\left[S_y(i) + 4 \cdot S_y(k) + S_y(j)\right]$$

$$I_{xy} = \sum_{n=1}^{Seg} -\frac{\Delta Y_k}{6}\left[S_x(i) + 4 \cdot S_x(k) + S_x(j)\right]$$

$$I_{yx} = \sum_{n=1}^{Seg} -\frac{\Delta X_k}{6}\left[S_y(i) + 4 \cdot S_y(k) + S_y(j)\right] \qquad (26)$$

Note: Seg. = total number of segment

Step 3 - Define normal stress (σ_x or σ_y) distribution due to wave and still water bending moments

Equation (16) shows that pure bending without twist on cross section could be divided into the vertical and horizontal normal stresses distribution. Where the normal stresses distribution on each segment described as follows:

$$W_{xmax} = \max\left[\frac{I_{xx}}{H - Y_c}, \frac{I_{xx}}{Y_c}\right] \qquad \sigma_{xmax} = \frac{M_V}{W_{min}}$$

$$W_{xmin} = \min\left[\frac{I_{xx}}{H - Y_c}, \frac{I_{xx}}{Y_c}\right] \qquad (27)$$

$$\sigma_{xmin} = \frac{M_V}{W_{max}} \quad \sigma_{x,i} = y_i \cdot \frac{(\sigma_{xmax} - \sigma_{xmin})}{H} + \sigma_{xmin}$$

$$\sigma_{x,j} = y_j \cdot \frac{(\sigma_{xmax} - \sigma_{xmin})}{H} + \sigma_{xmin}$$

$$\sigma_{x,k} = y_k \cdot \frac{(\sigma_{xmax} - \sigma_{xmin})}{H} + \sigma_{xmin}$$

$$W_{xmax} = \max\left[\frac{I_{yy}}{0.5 \cdot B - X_c}, \frac{I_{yy}}{0.5 \cdot B + X_c}\right] \quad \sigma_{ymax} = \frac{M_H}{W_{ymin}}$$

$$W_{xmin} = \min\left[\frac{I_{yy}}{0.5 \cdot B - X_c}, \frac{I_{yy}}{0.5 \cdot B + X_c}\right] \quad \sigma_{ymin} = \frac{M_H}{W_{ymax}}$$

$$\sigma_{y,i} = (x_i - 0.5 \cdot B) \cdot \frac{(\sigma_{ymax} - \sigma_{ymin})}{-B} + \sigma_{ymin}$$

$$\sigma_{y,j} = (x_j - 0.5 \cdot B) \cdot \frac{(\sigma_{ymax} - \sigma_{ymin})}{-B} + \sigma_{ymin} \qquad (28)$$

$$\sigma_{y,k} = (x_k - 0.5 \cdot B) \cdot \frac{(\sigma_{ymax} - \sigma_{ymin})}{-B} + \sigma_{ymin}$$

4. NUMERICAL FORM FOR WARPING NORMAL STRESS DISTRIBUTION

The numerical form of warping stress derived from equations on sub section 2.2. That subsection shows double sectorial area on cross section divided to the open and closed section. Therefore, if the multi cell or combination section were found it needs to be cut on its close section in order to create an open section for easier calculation. Then the integration along all of segments could be done. Finally, correction would be done on each segment of closed section to remove the slip effect due to cutting. Steps by step form of numerical calculation of warping stresses distribution were given below:

Step 1 - Define double sectorial area (DSA) with respect to (w.r.t) neutral axis (centroid) for open section (ω_{oNA})

The calculation of DSA could be done on open section which is based on Equation (18), therefore a numerical form is described as follows.

$$\omega_{oNA} = h_{pNA} \cdot L_k \quad ;\text{with } (x'_k = x_k - X_c \, , y'_k = y_k - Y_c)$$

$$\omega_{oNA} = x'_k \cdot \Delta Y_k - y'_k \cdot \Delta X_k \tag{29}$$

Step 2 – Define the DSA for closed section (ω_2)

In this step only the DSA of closed section will be calculated. Where that numerical form is derived from Equation (19):

$$\omega_2 = 2 \cdot [\delta] \cdot L_k / t \tag{30}$$

Where δ is flexural matric coefficients of single cell or multi cell is formulated as follows:

$$[\delta] = A \cdot [K]^{-1}$$

Where,

A = area of each cell

$= \{[(x_1.y_2)+(x_2.y_3)+(x_3.y_n)+(x_n.y_1)] - [(y_1.x_2)+(y_2.x_3)+(y_3.x_n)+(y_n.x_1)]\}/2$ (31)

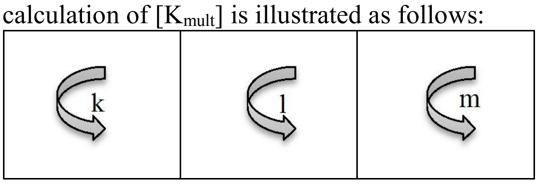
[K] = [K_{sing}] or [K_{mult}]

[K_{sing}] = for single cell

$= \sum_{n=i}^{cell} \frac{Lk_i}{t_i}$;n is number of segment in cell (32)

[K_{mult}] = for multi cell

for calculation of [K_{mult}] is illustrated as follows:



$$[K_{mult}] = \begin{bmatrix} \oint \frac{ds}{t_k} & -\oint \frac{ds}{t_{k,l}} & 0 \\ & \oint \frac{ds}{t_l} & -\oint \frac{ds}{t_{l,m}} \\ \text{symm} & & \oint \frac{ds}{t_m} \end{bmatrix} \tag{33}$$

Step 3 - Define the warping product of inertia about x and y axes ($I_{\omega x}$ and $I_{\omega y}$)

Before determined centre of twist on cross section, calculation of warping product of inertia w.r.t. x and y axes shall be done. Those calculations are applied to open and closed section with the numerical form is obtained from Equation (22):

$$I_{\omega xo} = \sum_{n=1}^{Seg} \frac{\omega_{oNA}}{6} \left[S_x(i) + 4 \cdot S_x(k) + S_x(j) \right]$$

$$I_{\omega yo} = \sum_{n=1}^{Seg} \frac{\omega_{oNA}}{6} \left[S_y(i) + 4 \cdot S_y(k) + S_y(j) \right]$$

$$I_{\omega xc} = \sum_{n=1}^{Seg} \frac{\omega_2}{6} \left[S_x(i) + 4 \cdot S_x(k) + S_x(j) \right]$$

$$I_{\omega yc} = \sum_{n=1}^{Seg} \frac{\omega_2}{6} \left[S_y(i) + 4 \cdot S_y(k) + S_y(j) \right] \tag{34}$$

Step 4 - Define centre of twist (x_0, y_0)

By Equation (21) including of its components is obtained from previous step, centre of twist (x_0, y_0) on cross section can be calculated.

Step 5 - Define the DSA w.r.t centre of twist ($\dot{\omega}_0$)

After centre of twist is found, then the DSA of each segment on step 1 will be calculated with respect to it position. Therefore, formulations the new DSA is described as follows:

$$\omega_o = h_{po} \cdot L_k$$

$$\omega_o = \Delta Y_k \left(x'_k - x_o \right) - \Delta X_k \left(y'_k - y_o \right)$$

$$\omega_o = \omega - \left(x_o \cdot \Delta Y_k - y_o \cdot \Delta X_k \right) \tag{35}$$

If a cross section comprised of multi cell or combination, value of $\dot{\omega}_0$ on each segment of closed section shall be corrected with $\dot{\omega}_2$. Hence, the formulation is given by:

$$\omega c = \omega_0 - \omega_2 \tag{36}$$

Then values of ωc are distributed along a segment nodal i, k, j which have two directions: (a) ascending flow at main branch, (b) descending flow at segment branch. Where those formulations are given by:

$$\left.\begin{array}{l} \omega_c, i_{initial} = 0 \text{ (at starting flow)} \\ \omega_c(i) = \omega_c(j) \\ \omega_c(k) = \dot{\omega}_c(i) + (0.5 \cdot \omega_c) \\ \omega_c(j) = \dot{\omega}_c(i) + \dot{\omega}_c \end{array}\right\} \text{ascending (flow)}$$

$$\tag{37}$$

$$\left.\begin{array}{l} \omega_c(j) = \omega_c(j_{asc}) \text{ or } \omega c(i_{desc}) \\ \omega_c(k) = \omega_c(j) - (0.5 \cdot \omega_c) \\ \omega_c(i) = \omega_c(j) - \omega_c \end{array}\right\} \text{descending (path flow)}$$

Step 6 - Define the warping function (ω)

Based on Equation (20) and (17) can be calculated normalized unit warping (ω_n) and warping function ($\dot{\omega}$) with formulations as follows:

$$\omega_n = \frac{\sum_{n=1}^{Seg} \frac{A_k}{6} \left[\omega_c(i) + 4 \cdot \omega_c(k) + \omega_c(j) \right]}{\sum A_k} \tag{38}$$

Therefore, warping distributions along a segment are:

$$\begin{aligned} \omega(i) &= \omega_c(i) - \omega_n \\ \omega(k) &= \omega_c(k) - \omega_n \\ \omega(j) &= \omega_c(j) - \omega_n \end{aligned} \tag{39}$$

Step 7 - Define the warping moment of inertia ($I_{\dot{\omega}\dot{\omega}}$)

Numerical form for calculation of warping moment of inertia ($I_{\dot{\omega}\dot{\omega}}$) is based on Equation (22) which is described as follows:

$$I_{\omega\omega} = \sum_{n=1}^{Seg} \frac{A_k}{6}\left[\omega(i) + 4\cdot\omega(k) + \omega(j)\right] \quad (40)$$

Step 8 - Define of warping normal stress ($\sigma_{\dot{\omega}}$) distribution
Equation (23) shows that bending subject to pure twist on cross section can be distributed on each segment which is described as follows:

$$\sigma_{\omega}(i) = \frac{M_{\omega}\cdot\omega(i)}{I_{\omega\omega}} \qquad \sigma_{\omega}(k) = \frac{M_{\omega}\cdot\omega(k)}{I_{\omega\omega}}$$

$$\sigma_{\omega}(j) = \frac{M_{\omega}\cdot\omega(j)}{I_{\omega\omega}} \quad (41)$$

5. NUMERICAL STUDY

Ship cross section with a combination of single open and closed (cell) even multi cell is applied to describe the proposed numerical procedure. Which its principal dimension is showed in Table 1 including with detailed model is represented in Figure 1

Table 1: Principal dimension of the sample model

Criteria	Dimension	
Length, between perpendiculars	110	m
Length, between waterlines	110	m
Length, scantling	106.7	m
Breadth, moulded	20	m
Depth, moulded	15	m
Draft, summer extreme	10	m
Speed	6	kn
Block coefficient	0.9	

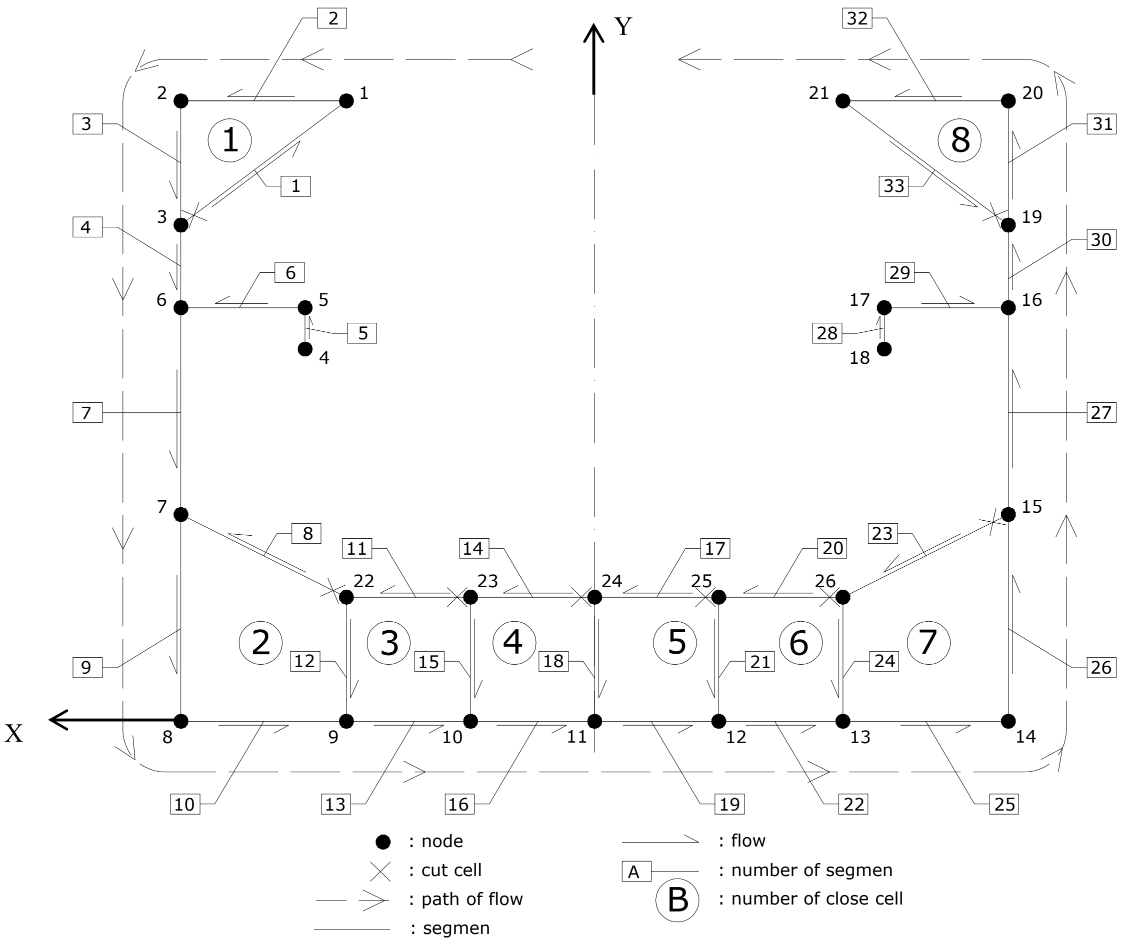


Figure 1: Sample model

The sample model on Figure 2 is divided into node (●) and segment (——). One segment only has two nodes i and j, hence it called flat segment. Therefore, for circular part of the cross section can be divided into many small of flat segment. Then define open and closed section with give a name on the closed section although it single or multi cell. For the closed section will be conducted a cutting (cut cell) to create an open section which is only one cut cell is permitted for each closed section (cell). The number of cut cell and cell shall be the same and for multi cell a cut cell shall be created an open section with a shortest branch. Then define a main flow on the cross section which it has the same pattern for all section. Starting point of main flow is the upper end of cross section which it coincides with centre line (CL) or closest to CL. Then flow along outer boundary of the cross section with counter clockwise direction (portside - starboard) until meet with the starting point or upper end on starboard side. Thus, all of segment inside outer boundary can be defined as branch in which have flows with starting point on all of free end of section. These flow is called a path flow which it flows toward the branching point on the outer boundary of cross section.

Step (1)

Start calculation is initiated with input nodal coordinates (Xi,Yi) and (Xj,Yj) of each segment with respect to an arbitrary coordinate system (X,Y), its thickness (t) and connectivity with other elements. Then determine the coordinate of midpoint length and area of each segment:

$$X_k = \frac{X_i + X_j}{2} \qquad ; \Delta X_k = X_j - X_i$$

$$Y_k = \frac{Y_i + Y_j}{2} \qquad ; \ \Delta Y_k = Y_j - Y_i$$

$$L_k = \sqrt{\Delta X^2 + \Delta Y^2} \qquad ; A_k = L_k \cdot t$$

The detail calculation of centre of gravity including its results is showed in Table 2.

Table 2. Calculation of centre of gravity

Node ID	X [m]	Y [m]	Seg.	Node i	Node j	thick. [m]	[xi]	[yi]	[xj]	[yj]	Xk [m]	Yk [m]	ΔXk [m]	ΔYk [m]	Lk [m]	Ak [m²]	Xk.Ak [m³]	Yk.Ak [m³]	[xk'] [m]	[yk'] [m]
1	-6	15	1	3	1	0.015	-10	12	-6	15	-8	14	4	3	5.00	0.075	-0.60	1.01	-8.0	7.7
2	-10	15	2	1	2	0.015	-6	15	-10	15	-8	15	-4	0	4.00	0.060	-0.48	0.90	-8.0	9.2
3	-10	12	3	2	3	0.015	-10	15	-10	12	-10	14	0	-3	3.00	0.045	-0.45	0.61	-10.0	7.7
4	-7	9	4	3	6	0.015	-10	12	-10	10	-10	11	0	-2	2.00	0.030	-0.30	0.33	-10.0	5.2
5	-7	10	5	4	5	0.015	-7	9	-7	10	-7	10	0	1	1.00	0.015	-0.11	0.14	-7.0	3.7
6	-10	10	6	5	6	0.015	-7	10	-10	10	-9	10	-3	0	3.00	0.045	-0.38	0.45	-8.5	4.2
7	-10	5	7	6	7	0.015	-10	10	-10	5	-10	8	0	-5	5.00	0.075	-0.75	0.56	-10.0	1.7
8	-10	0	8	22	7	0.01	-6	3	-10	5	-8	4	-4	2	4.47	0.045	-0.36	0.18	-8.0	-1.8
9	-6	0	9	7	8	0.015	-10	5	-10	0	-10	3	0	-5	5.00	0.075	-0.75	0.19	-10.0	-3.3
10	-3	0	10	8	9	0.015	-10	0	-6	0	-8	0	4	0	4.00	0.060	-0.48	0.00	-8.0	-5.8
11	0	0	11	23	22	0.01	-3	3	-6	3	-5	3	-3	0	3.00	0.030	-0.14	0.09	-4.5	-2.8
12	3	0	12	22	9	0.02	-6	3	-6	0	-6	2	0	-3	3.00	0.060	-0.36	0.09	-6.0	-4.3
13	6	0	13	9	10	0.015	-6	0	-3	0	-5	0	3	0	3.00	0.045	-0.20	0.00	-4.5	-5.8
14	10	0	14	24	23	0.01	0	3	-3	3	-2	3	-3	0	3.00	0.030	-0.05	0.09	-1.5	-2.8
15	10	5	15	23	10	0.02	-3	3	-3	0	-3	2	0	-3	3.00	0.060	-0.18	0.09	-3.0	-4.3
16	10	10	16	10	11	0.015	-3	0	0	0	-2	0	3	0	3.00	0.045	-0.07	0.00	-1.5	-5.8
17	7	10	17	25	24	0.01	3	3	0	3	2	3	-3	0	3.00	0.030	0.05	0.09	1.5	-2.8
18	7	9	18	24	11	0.02	0	3	0	0	0	2	0	-3	3.00	0.060	0.00	0.09	0.0	-4.3
19	10	12	19	11	12	0.015	0	0	3	0	2	0	3	0	3.00	0.045	0.07	0.00	1.5	-5.8
20	10	15	20	26	25	0.01	6	3	3	3	5	3	-3	0	3.00	0.030	0.14	0.09	4.5	-2.8
21	6	15	21	25	12	0.02	3	3	3	0	3	2	0	-3	3.00	0.060	0.18	0.09	3.0	-4.3
22	-6	3	22	12	13	0.015	3	0	6	0	5	0	3	0	3.00	0.045	0.20	0.00	4.5	-5.8
23	-3	3	23	15	26	0.01	10	5	6	3	8	4	-4	-2	4.47	0.045	0.36	0.18	8.0	-1.8
24	0	3	24	26	13	0.02	6	3	6	0	6	2	0	-3	3.00	0.060	0.36	0.09	6.0	-4.3
25	3	3	25	13	14	0.015	6	0	10	0	8	0	4	0	4.00	0.060	0.48	0.00	8.0	-5.8
26	6	3	26	14	15	0.015	10	0	10	5	10	3	0	5	5.00	0.075	0.75	0.19	10.0	-3.3
			27	15	16	0.015	10	5	10	10	10	8	0	5	5.00	0.075	0.75	0.56	10.0	1.7
			28	18	17	0.015	7	9	7	10	7	10	0	1	1.00	0.015	0.11	0.14	7.0	3.7
			29	17	16	0.015	7	10	10	10	9	10	3	0	3.00	0.045	0.38	0.45	8.5	4.2
			30	16	19	0.015	10	10	10	12	10	11	0	2	2.00	0.030	0.30	0.33	10.0	5.2
			31	19	20	0.015	10	12	10	15	10	14	0	3	3.00	0.045	0.45	0.61	10.0	7.7
			32	20	21	0.015	10	15	6	15	8	15	-4	0	4.00	0.060	0.48	0.90	8.0	9.2
			33	21	19	0.015	6	15	10	12	8	14	4	-3	5.00	0.075	0.60	1.01	8.0	7.7
													0.00	**-13.00**	**111.94**	**1.65**	**0.00**	**9.55**		

Table 2 give information about total number of ΣAk = 1.65 m^2, $\Sigma(X_k \cdot A_k) = 0.00$ and $\Sigma(Y_k \cdot A_k) = 9.55$. The sum of $\Sigma(X_k \cdot A_k) = 0.00$ because the form of cross section is symmetrical. Values of X_k and Y_k can be used to determined centre of gravity (X_c, Y_c). Centre of gravity can be calculated using equation (25). Therefore, coordinate centre of gravity is found (0.00, 5.79). Value of X_c is zero because the form of cross section is symmetrical like mentioned before. In other hand calculation of xk' and yk' as components calculation the DSA at the further step is conducted. Those values can be obtained using Equation (29)

Step (2)

After the centre of gravity is determined, the next step is determined moment of inertia with respect to centre of gravity (neutral axes). Using numerical procedure in Equation (26) these values can be obtained. According to these procedure moment of inertia are calculated by static moment (Sx or Sy) on a cut of portion of the cross section for points i, j and k of each segment. Then Sx/Sy will be distributed along a segment into nodal coordinate i, k, j. Hence, using Equation (26) distribution of Sx/Sy will be integrated on each along segment length of ship cross section. The result of moment of inertia calculation described on Table 3. These table give the information about moment of inertia characteristics of cross section. Where the values of moment of inertia x with respect to neutral axis ($I_{NA\text{-}x}$ = 48.03), moment of inertia y with respect to neutral axis ($I_{NA\text{-}y}$ = 93.36) and product moment of inertia x and y ($I_{xy} = I_{yx} = 0.00$). Zero values of $I_{xy} = I_{yx}$ because of symmetrical form of cross section was evaluated.

Table 3. Moment of Inertia Calculation

Seg.	Node i	Node j	Sxi' [m^3]	Sxk' [m^3]	Sxj' [m^3]	P_{roduct}-x	I_{NA}-x [m^4]	I_{NA}-xy [m^4]	Syi' [m^3]	Syk' [m^3]	Syj' [m^3]	P_{roduct}-y	I_{NA}-y [m^4]	I_{NA}-yx [m^4]
1	3	1	0.00	0.26	0.58	1.622	-0.811	-1.08	0.00	-0.34	-0.60	-1.95	1.30	0.98
2	1	2	0.58	0.85	1.13	5.126	0.000	3.42	-0.60	-0.81	-1.08	-4.92	-3.28	0.00
3	2	3	1.13	1.32	1.48	7.892	3.946	0.00	-1.08	-1.31	-1.53	-7.83	0.00	-3.92
4	3	6	1.48	1.56	1.63	9.364	3.121	0.00	-1.53	-1.68	-1.83	-10.08	0.00	-3.36
5	4	5	0.00	0.03	0.06	0.159	-0.027	0.00	0.00	-0.05	-0.11	-0.32	0.00	0.05
6	5	6	0.06	0.15	0.25	0.902	0.000	0.45	-0.11	-0.28	-0.49	-1.71	-0.86	0.00
7	6	7	1.88	1.99	2.01	11.845	9.871	0.00	-2.32	-2.69	-3.07	-16.16	0.00	-13.46
8	22	7	0.00	-0.05	-0.08	-0.285	0.095	-0.19	0.00	-0.16	-0.36	-0.98	-0.66	0.33
9	7	8	1.93	1.85	1.68	11.008	9.173	0.00	-3.43	-3.80	-4.18	-22.80	0.00	-19.00
10	8	9	1.68	1.51	1.33	9.037	0.000	-6.02	-4.18	-4.45	-4.66	-26.61	17.74	0.00
11	23	22	0.00	-0.04	-0.08	-0.251	0.000	-0.13	0.00	-0.06	-0.14	-0.36	-0.18	0.00
12	22	9	-0.08	-0.19	-0.34	-1.185	-0.592	0.00	-0.14	-0.32	-0.50	-1.89	0.00	-0.95
13	9	10	0.99	0.86	0.73	5.165	0.000	-2.58	-5.15	-5.27	-5.35	-31.58	15.79	0.00
14	24	23	0.00	-0.04	-0.08	-0.251	0.000	-0.13	0.00	-0.01	-0.05	-0.09	-0.05	0.00
15	23	10	-0.08	-0.19	-0.34	-1.185	-0.592	0.00	-0.05	-0.14	-0.23	-0.81	0.00	-0.41
16	10	11	0.39	0.26	0.13	1.554	0.000	-0.78	-5.58	-5.63	-5.65	-33.74	16.87	0.00
17	25	24	0.00	-0.04	-0.08	-0.251	0.000	-0.13	0.00	0.03	0.05	0.18	0.09	0.00
18	24	11	-0.08	-0.19	-0.34	-1.185	-0.592	0.00	0.05	0.05	0.05	0.27	0.00	0.14
19	11	12	-0.21	-0.34	-0.47	-2.057	0.000	1.03	-5.60	-5.58	-5.53	-33.47	16.73	0.00
20	26	25	0.00	-0.04	-0.08	-0.251	0.000	-0.13	0.00	0.08	0.14	0.45	0.23	0.00
21	25	12	-0.08	-0.19	-0.34	-1.185	-0.592	0.00	0.14	0.23	0.32	1.35	0.00	0.68
22	12	13	-0.81	-0.94	-1.07	-5.668	0.000	2.83	-5.22	-5.13	-5.02	-30.77	15.38	0.00
23	15	26	0.00	-0.03	-0.08	-0.196	-0.065	-0.13	0.00	0.20	0.36	1.16	0.78	0.39
24	26	13	-0.08	-0.19	-0.34	-1.163	-0.582	0.00	0.36	0.54	0.72	3.23	0.00	1.61
25	13	14	-1.41	-1.59	-1.76	-9.518	0.000	6.35	-4.30	-4.09	-3.82	-24.47	16.31	0.00
26	14	15	-1.76	-1.93	-2.01	-11.488	9.574	0.00	-3.82	-3.44	-3.07	-20.66	0.00	17.21
27	15	16	-2.01	-1.99	-1.88	-11.845	9.871	0.00	-3.07	-2.69	-2.32	-16.16	0.00	13.46
28	18	17	0.00	0.03	0.06	0.159	-0.027	0.00	0.00	0.05	0.11	0.32	0.00	-0.05
29	17	16	0.06	0.15	0.25	0.902	0.000	-0.45	0.11	0.28	0.49	1.71	-0.86	0.00
30	16	19	-1.63	-1.56	-1.48	-9.364	3.121	0.00	-1.83	-1.68	-1.53	-10.08	0.00	3.36
31	19	20	-1.48	-1.32	-1.13	-7.892	3.946	0.00	-1.53	-1.31	-1.08	-7.83	0.00	3.92
32	20	21	-1.13	-0.85	-0.58	-5.126	0.000	-3.42	-1.08	-0.81	-0.60	-4.92	-3.28	0.00
33	21	19	-0.58	-0.26	0.00	-1.622	-0.811	1.08	-0.60	-0.34	0.00	-1.95	1.30	-0.97
							48.03	**0.00**					**93.36**	**0.00**

Step (3)

This steps to define of normal stress distribution due to hull girder wave vertical and horizontal bending moments. Distribution of hull girder load can be determined according to the BKI Rule for Hull, Section 5, 2014. These load distributions are showed in Figure 2 where its illustrated distribution of vertical wave bending moment in sagging and hogging condition (M_{wvHog}, M_{wvSag}), horizontal bending moment (M_{wH}), wave torsional moment (M_{wT}), maximum and minimum still water bending moment (S_{wmax}, S_{wmin}). For determination of normal stress distribution is used maximum values of M_{wvHog}, M_{wvSag} and $M_{wH.}$ Based on the Figure 2 those values are 313924.3 kNm, -323103 kNm, and 323179.7 kNm respectively. The results of normal stresses distribution are showed in Table 4.

Table 4 describe of normal stress distribution due to vertical wave hogging & sagging condition (σ_{hog}, σ_{sag}) and horizontal bending moment (σ_{bmh}). According to the Equation (27) and (28), those values distributed along segment nodal coordinate i, k, j. For the greatest values of σ_{hog} is occurred in the deck, but opposite response is occurred for σ_{sag} which it located at the bottom. In other hand the greatest values of σ_{bmh} is occurred on the side of ship.

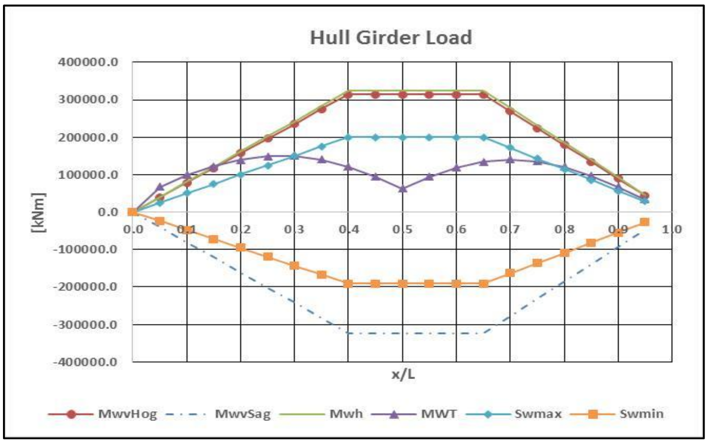


Figure 2: Distribution of Hull Girder Load (BKI Rules for Hull, Vol II)

Table 4. Normal stress distribution of σ_{hog}, σ_{sag}, and σ_{bmh}

Seg.	Node i	Node j	σhog-i	σhog-k	σhog-j	σsag-i	σsag-k	σsag-j	σbmh-i	σbmh-k	σbmh-j
1	3	1	40.58	50.39	60.19	-41.77	-51.86	-61.95	34.62	27.69	20.77
2	1	2	60.19	60.19	60.19	-61.95	-61.95	-61.95	20.77	27.69	34.62
3	2	3	60.19	50.39	40.58	-61.95	-51.86	-41.77	34.62	34.62	34.62
4	3	6	40.58	34.05	27.51	-41.77	-35.04	-28.31	34.62	34.62	34.62
5	4	5	20.97	24.24	27.51	-21.59	-24.95	-28.31	24.23	24.23	24.23
6	5	6	27.51	27.51	27.51	-28.31	-28.31	-28.31	24.23	29.42	34.62
7	6	7	27.51	11.17	-5.17	-28.31	-11.49	5.33	34.62	34.62	34.62
8	22	7	-18.25	-11.71	-5.17	18.78	12.05	5.33	20.77	27.69	34.62
9	7	8	-5.17	-21.52	-37.86	5.33	22.14	38.96	34.62	34.62	34.62
10	8	9	-37.86	-37.86	-37.86	38.96	38.96	38.96	34.62	27.69	20.77
11	23	22	-18.25	-18.25	-18.25	18.78	18.78	18.78	10.38	15.58	20.77
12	22	9	-18.25	-28.05	-37.86	18.78	28.87	38.96	20.77	20.77	20.77
13	9	10	-37.86	-37.86	-37.86	38.96	38.96	38.96	20.77	15.58	10.38
14	24	23	-18.25	-18.25	-18.25	18.78	18.78	18.78	0.00	5.19	10.38
15	23	10	-18.25	-28.05	-37.86	18.78	28.87	38.96	10.38	10.38	10.38
16	10	11	-37.86	-37.86	-37.86	38.96	38.96	38.96	10.38	5.19	0.00
17	25	24	-18.25	-18.25	-18.25	18.78	18.78	18.78	-10.38	-5.19	0.00
18	24	11	-18.25	-28.05	-37.86	18.78	28.87	38.96	0.00	0.00	0.00
19	11	12	-37.86	-37.86	-37.86	38.96	38.96	38.96	0.00	-5.19	-10.38
20	26	25	-18.25	-18.25	-18.25	18.78	18.78	18.78	-20.77	-15.58	-10.38
21	25	12	-18.25	-28.05	-37.86	18.78	28.87	38.96	-10.38	-10.38	-10.38
22	12	13	-37.86	-37.86	-37.86	38.96	38.96	38.96	-10.38	-15.58	-20.77
23	15	26	-5.17	-11.71	-18.25	5.33	12.05	18.78	-34.62	-27.69	-20.77
24	26	13	-18.25	-28.05	-37.86	18.78	28.87	38.96	-20.77	-20.77	-20.77
25	13	14	-37.86	-37.86	-37.86	38.96	38.96	38.96	-20.77	-27.69	-34.62
26	14	15	-37.86	-21.52	-5.17	38.96	22.14	5.33	-34.62	-34.62	-34.62
27	15	16	-5.17	11.17	27.51	5.33	-11.49	-28.31	-34.62	-34.62	-34.62
28	18	17	20.97	24.24	27.51	-21.59	-24.95	-28.31	-24.23	-24.23	-24.23
29	17	16	27.51	27.51	27.51	-28.31	-28.31	-28.31	-24.23	-29.42	-34.62
30	16	19	27.51	34.05	40.58	-28.31	-35.04	-41.77	-34.62	-34.62	-34.62
31	19	20	40.58	50.39	60.19	-41.77	-51.86	-61.95	-34.62	-34.62	-34.62
32	20	21	60.19	60.19	60.19	-61.95	-61.95	-61.95	-34.62	-27.69	-20.77
33	21	19	60.19	50.39	40.58	-61.95	-51.86	-41.77	-20.77	-27.69	-34.62

Step (4)
In these step is calculated warping function (ω) and warping moment of inertia ($I_{\omega\omega}$) of cross section. Calculation is done using numerical procedure that is represented in the Equation (29) up to (41). Based on those equation warping function has two types, namely warping for open and closed section. In this study the ship cross-section is a combination of open, single closed cell, and multi cell. Thus, for solving these issued will be cut the close cell (cut cell) of each section to create it an open section. These cut cell is showed in the Figure 2, from that figure can be seen that after applied a cut cell the ship cross section becomes fully open. After that, can be performed the calculation of DSA for the open section on the overall segment in the cross section ($\dot{\omega}_o$). Then calculated the DSA for single cell and multi cell ($\dot{\omega}_2$).In the calculation of $\dot{\omega}_2$ would be performed a calculation of matrices (δ). Where in the single cell can be solved directly using the Equation (31) and (33). In other hand for multi cell is used a flexural matric [K_{mult}] that represented the correlation of one segment which used into two cell or more (Table 5).

Table 5. Calculation of flexural matric coefficient

Cell	Seg.	Node i	Node j	[xi]	[yi]	[xj]	[yj]	A-i	ΣA	∫ ds/t	Σ K	Σ A/[K]	A/[K]-i
1	3	2	3	-10	15	-10	12	0	6	200.00	800.0	0.00750	0.00750
	2	1	2	-6	15	-10	15	0		266.67			0.00750
	1	3	1	-10	12	-6	15	6		333.33			0.00750
2	8	22	7	-6	3	-10	5	0	16	447.21	1197.2	0.01556	0.01556
	9	7	8	-10	5	-10	0	10		333.33			0.01556
	10	8	9	-10	0	-6	0	6		266.67			0.01556
	12	22	9	-6	3	-6	0	0		150.00			0.00196
3	11	23	22	-3	3	-6	3	0	9	300.00	800.0	0.01752	0.01752
	12	22	9	-6	3	-6	0	4.5		150.00			0.00196
	13	9	10	-6	0	-3	0	4.5		200.00			0.01752
	15	23	10	-3	3	-3	0	0		150.00			0.00037
4	14	24	23	0	3	-3	3	0	9	300.00	800.0	0.01789	0.01789
	15	23	10	-3	3	-3	0	4.5		150.00			0.00037
	16	10	11	-3	0	0	0	4.5		200.00			0.01789
	18	24	11	0	3	0	0	0		150.00			0.00000
5	17	25	24	3	3	0	3	0	9	300.00	800.0	0.01789	0.01789
	18	24	11	0	3	0	0	4.5		150.00			0.00000
	19	11	12	0	0	3	0	4.5		200.00			0.01789
	21	25	12	3	3	3	0	0		150.00			-0.00037
6	20	26	25	6	3	3	3	0	9	300.00	800.0	0.01752	0.01752
	21	25	12	3	3	3	0	4.5		150.00			-0.00037
	22	12	13	3	0	6	0	4.5		200.00			0.01752
	24	26	13	6	3	6	0	0		150.00			-0.00196
7	23	15	26	10	5	6	3	0	16	447.21	1197.2	0.01556	0.01556
	24	26	13	6	3	6	0	6		150.00			-0.00196
	25	13	14	6	0	10	0	10		266.67			0.01556
	26	14	15	10	0	10	5	0		333.33			0.01556
8	31	19	20	10	12	10	15	0	6	200.00	800.0	0.00750	0.00750
	32	20	21	10	15	6	15	6		266.67			0.00750
	33	21	19	6	15	10	12	0		333.33			0.00750

Matric [K]

	2	3	4	5	6	7
2	1197.21	-150.00	0	0	0	0
3	-150.00	800.00	-150	0	0	0
4	0	-150	800	-150	0	0
5	0	0	-150	800	-150	0
6	0	0	0	-150	800	-150
7	0	0	0	0	-150	1197

Matric $[K]^{-1}$ **Area {A}** **$\{A\}\times[K]^{-1}$**

$$
\begin{vmatrix}
8.56E{-}04 & 1.67E{-}04 & 3.24E{-}05 & 6.31E{-}06 & 1.21E{-}06 & 1.52E{-}07 \\
1.67E{-}04 & 1.33E{-}03 & 2.59E{-}04 & 5.03E{-}05 & 9.66E{-}06 & 1.21E{-}06 \\
3.24E{-}05 & 2.59E{-}04 & 1.35E{-}03 & 2.62E{-}04 & 5.03E{-}05 & 6.31E{-}06 \\
6.31E{-}06 & 5.03E{-}05 & 2.62E{-}04 & 1.35E{-}03 & 2.59E{-}04 & 3.24E{-}05 \\
1.21E{-}06 & 9.66E{-}06 & 5.03E{-}05 & 2.59E{-}04 & 1.33E{-}03 & 1.67E{-}04 \\
1.52E{-}07 & 1.21E{-}06 & 6.31E{-}06 & 3.24E{-}05 & 1.67E{-}04 & 8.56E{-}04
\end{vmatrix}
\times
\begin{vmatrix} 16 \\ 9 \\ 9 \\ 9 \\ 9 \\ 16 \end{vmatrix}
=
\begin{array}{c|l}
\begin{vmatrix} 0.0156 \\ 0.0175 \\ 0.0179 \\ 0.0179 \\ 0.0175 \\ 0.0156 \end{vmatrix} & \begin{array}{l} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}
\end{array}
$$

In Table 5 is described of detail calculation of flexural matric in which multi cell is indicated with a red mark. This is based on to the Figure 1 where it can be seen that cell 1 and 8 are single cell while the cell of 2 - 7 is multi cell. Therefore, those can be assembled a flexural matrix coefficient for cell 2 – 7 by Equation (32). Hence, the result of multiplying matric $\{A\}*[K]^{-1}$ is matric (δ), then using Equation (30) can be obtained $\dot{\omega}_2$.

Value of $\dot{\omega}_{oNA}$ is calculated with respect to centre of gravity. But to obtain a warping moment of inertia ($I_{\dot{\omega}\dot{\omega}}$) must be determined the location of the centre of twist in advance by using Equation (21). Components to calculate the centre of twist are warping product of inertia about x and y axes in which it can be calculated using Equation (34). Those calculations have two values, namely $I\dot{\omega}xo$ (open) and $I\dot{\omega}xc$ (closed). Therefore the real value of warping product of Inertia ($I_{\dot{\omega}x}$ or $I_{\dot{\omega}y}$) is the result of reduction $I_{\dot{\omega}xo}$ by $I_{\dot{\omega}xc}$ (Equation (22)). Thus using Equation (21) distances of centre of twist (x_0, y_0) from centre of gravity is obtained where these value is (0.00, -11.61).

After obtained (x_0, y_0) can be calculated of $\dot{\omega}_o$ using Equation (35). Then will be obtained $\dot{\omega}_c$ using Equation (36) which its distribution shall be following the direction of flow (Figure 1). Therefore, for each path flow that has an opposite direction to the flow direction shall be using formulation of descending distribution in Equation (37). With obtained of distribution $\dot{\omega}_c$ the normalized warping function ($\dot{\omega}_n$) and warping function ($\dot{\omega}$) distribution can be calculated using Equation (38) and (39) respectively. Finally, using those values can be determined the warping moment of inertia ($I_{\omega\omega}$) with Equation (40). The results of all those quantities described in Table 6.

Table 6. Calculation of warping function ($\dot{\omega}_n$) and warping moment of inertia ($I_{\omega\omega}$)

Seg.	Node i	Node j	$\dot{\omega}_{oNA}$ [m^2]	{A}/[K]-i	$\dot{\omega}_2$ [m^2]	$I\dot{\omega}x_o$ [m^5]	$I\dot{\omega}x_c$ [m^5]	$I\dot{\omega}y_o$ [m^5]	$I\dot{\omega}y_c$ [m^5]	$\dot{\omega}_0$ [m^2]	$\dot{\omega}_c$ [m^2]	$\dot{\omega}_c$-i	$\dot{\omega}_c$-k	$\dot{\omega}_c$-j	$\dot{\omega}n$ [m^2]	ώi	ώk	ώj	Iώώ [m^6]
1	3	1	-54.83	0.0075	5.00	14.82	-1.35	-17.82	1.63	-101.27	-106.27	0.00	-53.13	-106.27	-2.42	-29.00	-82.13	-135.26	576.47
2	1	2	36.83	0.0075	4.00	-31.47	-3.42	30.20	3.28	83.27	79.27	-106.27	-66.63	-27.00	-2.42	-135.26	-95.63	-56.00	580.11
3	2	3	30.00	0.0075	3.00	-39.46	-3.95	39.15	3.92	30.00	27.00	-27.00	-13.50	0.00	-0.37	-56.00	-42.50	-29.00	84.00
4	3	6	20.00	0.0000	0.00	-31.21	0.00	33.60	0.00	20.00	20.00	0.00	10.00	20.00	0.18	-29.00	-19.00	-9.00	11.83
5	4	5	-7.00	0.0000	0.00	0.19	0.00	-0.37	0.00	-7.00	-7.00	-20.45	-23.95	**-27.45**	-0.22	-49.45	-52.95	-56.45	42.11
6	5	6	12.63	0.0000	0.00	-1.90	0.00	3.60	0.00	47.45	47.45	-27.45	-3.73	**20.00**	-0.10	-56.45	-32.72	-9.00	56.62
7	6	7	50.00	0.0000	0.00	-98.71	0.00	134.63	0.00	50.00	50.00	20.00	45.00	70.00	2.05	-9.00	16.00	41.00	34.84
8	22	7	-23.17	0.0156	13.92	-1.10	0.66	-3.80	2.28	23.27	9.35	60.65	65.32	**70.00**	1.77	31.65	36.33	41.00	59.35
9	7	8	50.00	0.0156	10.37	-91.73	-19.03	190.01	39.42	50.00	39.63	70.00	89.81	109.63	4.08	41.00	60.82	80.63	287.22
10	8	9	23.17	0.0156	8.30	-34.89	-12.50	102.75	36.81	-23.27	-31.57	109.63	93.84	78.06	3.41	80.63	64.85	49.07	257.30
11	23	22	-8.37	0.0175	10.51	-0.35	0.44	-0.50	0.63	26.45	15.94	44.71	52.68	**60.65**	0.96	15.72	23.69	31.65	17.46
12	22	9	18.00	0.0020	0.59	3.55	0.12	5.67	0.19	18.00	17.41	60.65	69.36	**78.06**	2.52	31.65	40.36	49.07	99.25
13	9	10	17.37	0.0175	7.01	-14.96	-6.03	91.44	36.89	-17.45	-24.46	78.06	65.83	53.60	1.80	49.07	36.84	24.61	63.30
14	24	23	-8.37	0.0179	10.73	-0.35	0.45	-0.13	0.16	26.45	15.72	29.00	36.85	**44.71**	0.67	0.00	7.86	15.72	2.47
15	23	10	9.00	0.0004	0.11	1.78	0.02	1.22	0.01	9.00	8.89	44.71	49.16	**53.60**	1.79	15.72	20.16	24.61	24.78
16	10	11	17.37	0.0179	7.16	-4.50	-1.85	97.69	40.24	-17.45	-24.61	53.60	41.30	29.00	1.13	24.61	12.30	0.00	9.08
17	25	24	-8.37	0.0179	10.73	-0.35	0.45	0.25	-0.32	26.45	15.72	13.28	21.14	**29.00**	0.38	-15.72	-7.86	0.00	2.47
18	24	11	0.00	0.0000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	29.00	29.00	**29.00**	1.05	0.00	0.00	0.00	0.00
19	11	12	17.37	0.0179	7.16	5.96	2.45	96.91	39.91	-17.45	-24.61	29.00	16.69	4.39	0.46	0.00	-12.30	-24.61	9.08
20	26	25	-8.37	0.0175	10.51	-0.35	0.44	0.63	-0.79	26.45	15.94	-2.66	5.31	**13.28**	0.10	-31.65	-23.69	-15.72	17.46
21	25	12	-9.00	-0.0004	-0.11	-1.78	-0.02	2.03	0.02	-9.00	-8.89	13.28	8.83	**4.39**	0.32	-15.72	-20.16	-24.61	24.78
22	12	13	17.37	0.0175	7.01	16.41	6.62	89.09	35.94	-17.45	-24.46	4.39	-7.84	-20.07	-0.21	-24.61	-36.84	-49.07	63.30
23	15	26	-23.17	0.0156	13.92	-0.76	0.45	4.49	-2.70	23.27	9.35	-12.01	-7.33	**-2.66**	-0.20	-41.00	-36.33	-31.65	59.35
24	26	13	-18.00	-0.0020	-0.59	-3.49	-0.11	9.68	0.32	-18.00	-17.41	-2.66	-11.36	**-20.07**	-0.41	-31.65	-40.36	-49.07	99.25
25	13	14	23.17	0.0156	8.30	36.75	13.16	94.46	33.84	-23.27	-31.57	-20.07	-35.85	-51.64	-1.30	-49.07	-64.85	-80.63	257.30
26	14	15	50.00	0.0156	10.37	95.74	19.86	172.13	35.71	50.00	39.63	-51.64	-31.82	-12.01	-1.45	-80.63	-60.82	-41.00	287.22
27	15	16	50.00	0.0000	0.00	98.71	0.00	134.63	0.00	50.00	50.00	-12.01	12.99	37.99	0.59	-41.00	-16.00	9.00	34.84
28	18	17	7.00	0.0000	0.00	-0.19	0.00	-0.37	0.00	7.00	7.00	78.44	81.94	**85.44**	0.75	49.45	52.95	56.45	42.11
29	17	16	-12.63	0.0000	0.00	1.90	0.00	3.60	0.00	-47.45	-47.45	85.44	61.72	**37.99**	1.68	56.45	32.72	9.00	56.62
30	16	19	20.00	0.0000	0.00	31.21	0.00	33.60	0.00	20.00	20.00	37.99	47.99	57.99	0.87	9.00	19.00	29.00	11.83
31	19	20	30.00	0.0075	3.00	39.46	3.95	39.15	3.92	30.00	27.00	57.99	71.49	84.99	1.95	29.00	42.50	56.00	84.00
32	20	21	36.83	0.0075	4.00	31.47	3.42	30.20	3.28	83.27	79.27	84.99	124.63	164.26	4.53	56.00	95.63	135.26	580.11
33	21	19	-54.83	0.0075	5.00	-14.82	1.35	-17.82	1.62	-101.27	-106.27	164.26	111.13	57.99	5.05	135.26	82.13	29.00	576.47
						5.58	**5.58**	**1399.99**	**316.20**						**29.00**		**135.26**		**4412.41**

Table 6 is showed the results application of numerical procedures to determine warping function and warping moment of inertia. Where the results indicated that value of ω_2 will be zero on each segment of the open cross section. These means only closed and multi cell will be corrected by ω_2 in order to eliminate the effect of slip due to applying a cut cell. In this case the corrections are applied on ω_o which calculated warping functions with respect to centre of twist. After correction can be generated ω_c values.

From the distributions results of ω_c in Table 6 can be known where the segment members that followed the path flow (descending) rules. If distribution procedure that used before define of the start of the integration is at each free end of the open section but in ω_c distribution initial integration has only one. In these case node 3 from segment 1 is initial node for integration where these indicated with zero value for distribution $\omega_{c,i}$ and its integration direction is called flow (ascending). Therefore, when the segment 5 (4-5) and 6 (5-6) have an opposite direction to flow shall be reversed to be 6 (6-5), 5 (5-4). Then in Table 5 is showed at the end of segment 4 distribution value for wc,j = 20 this is same value for wc,j in segment 6 (ω_cj, 6 = ω_cj, 4). With integration of distribution ω_c (ω_{ci}, ω_{ck}, ω_{cj}) can be obtained sum of $\dot{\omega}_n$ (29.00) as indicated in the Table 6. After that, can be calculated $\dot{\omega}$ distribution on each segment and its integration is generated $I\dot{\omega}\dot{\omega}$ (4412.41).

Step (5)
In the last step is presented calculations of warping normal stress distribution (σ_ω) on all segments of the ship cross section. From figure 2 can be obtained torsional moment 63386.43 kN.m where values of the warping function distribution (ω_i, ω_k, ω_j) and warping moment of inertia ($I\omega\omega$) are obtained from Table 6. Using those values will be generated distribution of σ_ω by applying of Equation (41). Where the distribution of σ_ω is described details in Table 7.

Table 7. Warping stress distribution, σ_ω

Seg.	Node i	Node j	σώ-i [N/mm²]	σώ-j [N/mm²]	σώ-k [N/mm²]
1	3	1	13.494	38.220	62.947
2	1	2	62.947	44.503	26.059
3	2	3	26.059	19.776	13.494
4	3	6	13.494	8.840	4.186
5	4	5	23.011	24.640	26.268
6	5	6	26.268	15.227	4.186
7	6	7	4.186	-7.448	-19.082
8	22	7	-14.731	-16.906	-19.082
9	7	8	-19.082	-28.303	-37.523
10	8	9	-37.523	-30.178	-22.834
11	23	22	-7.314	-11.022	-14.731
12	22	9	-14.731	-18.782	-22.834
13	9	10	-22.834	-17.142	-11.451
14	24	23	0.000	-3.657	-7.314
15	23	10	-7.314	-9.383	-11.451
16	10	11	-11.451	-5.726	0.000
17	25	24	7.314	3.657	0.000
18	24	11	0.000	0.000	0.000
19	11	12	0.000	5.726	11.451
20	26	25	14.731	11.022	7.314
21	25	12	7.314	9.383	11.451
22	12	13	11.451	17.142	22.834
23	15	26	19.082	16.906	14.731
24	26	13	14.731	18.782	22.834
25	13	14	22.834	30.178	37.523
26	14	15	37.523	28.303	19.082
27	15	16	19.082	7.448	-4.186
28	18	17	-23.011	-24.640	-26.268
29	17	16	-26.268	-15.227	-4.186
30	16	19	-4.186	-8.840	-13.494
31	19	20	-13.494	-19.776	-26.059
32	20	21	-26.059	-44.503	-62.947
33	21	19	-62.947	-38.220	-13.494
				62.95	

In the Table 7, it can be sseen the value of distribution σ_ω on each segment of ship cross section. Where the distribution value of the σ_ω can be indicated that the cross section of the port side was twisted forward (+ value) and backward (- value) in the starboard side with the same magnitude. The greatest value of σ_ω either forward or backward occurred on the ship's deck. This is because that section is an opening part with the longest position from the centre of twist. So with the wide openings ship location of centre of twist will be shifted down from the bottom line of ship until reached equilibrium between external and internal force. Unlike the case if the cross-section is a square box, the location of the centre of twist will be located in the middle of the object.

6. VALIDATION

Validation is done virtually by comparing the graphic results of the response stress distributions of the ship cross section with commercial software. In this case the commercial software is the GL Poseidon 2014. The comparison is conducted on response stress distribution due to moment of hogging, sagging, horizontal and torsional. Where successive comparison resulted is illustrated in Figure 3(a)-(d).

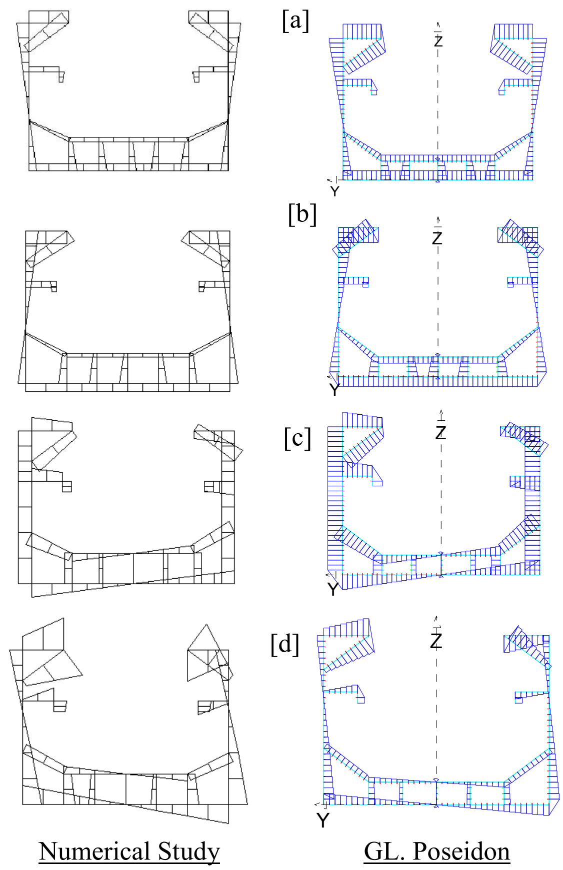


Figure 3:Virtually comparison for (a) σ_{hog}, (b) σ_{sag}, (c) σ_{bmh}, and (d) σ_ω distribution

In the Figure 3(a) stress response is occurred globally similar between the results of numerical studies and Poseidon. Which it the maximum and minimum stress occurred on the deck and bottom of ship respectively. While on the ship's side stress is distributed linearly which zero value at the neutral axis. Similar response is happened in Figure 3(b) but the maximum stress evenly occured on the bottom line and the minimum on the deck of the ship. And for Figure 3(c) is indicated a maximum stress occurred evenly on the portside and minimum on the starboard which a linear response is occurred in the deck and bottom of the ship. Finally, in Figure 3(d) warping stress response is indicated that comparison results between of numerical study and GL Poseidon have almost the same behaviour which ship cross section twisted forward on the portside and backward on starboard of ship.

All comparison result showed that numerical procedure is verified to apply on ship cross section. However, in reality ships structures are composed with combination of plate and stiffener (stiffened plate). Therefore, to improve the result for more realistic and accurate is necessary to developed numerical procedure for ship cross section which accommodating plate and stiffened plate element.

7. CONCLUSION

This study was presented a Simplified numerical procedure that has been applied to combination ship cross section including its examination using GL Poseidon 2014. From these results several important points can be concluded:

a. This study provides a detailed procedure for computing the normal stress distribution of ship's cross section with combination of single open, closed section and multi cell. This procedure is provided to help practicing engineers calculate (σ_{hog}, σ_{sag}, σ_{bmh}, σ_{ω}) values for ship cross section including hull girder load from BKI Rule for Hull II, Section 5, 2014.
b. The developed procedure could be applied to any platform of software calculation, e.g. Ms Excel, Mathcad, Matlab, etc..
c. Numerical procedure for shear stress distribution also need to be developed considering the normal stress and the shear stress are an important component in the determination of local structural failure due to hull girder loads.
d. To approach to more realistic result, developing a simplified numerical procedure calculation for normal stress distribution of combination ship cross section including stiffened plate element is necessary.

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